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Gamma-Radiation and its Relation to Nuclear Structure

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A formula has been obtained which enables the calculation of the wave-lengths of γ-radiation from the nucleus. Such calculations have been made for fourteen different radio-active elements and the observed and calculated λ's compared. The largest discrepancy is about twelve percent. γ-ray series which check the observed γ-ray wave-lengths to less than five percent have been calculated for AcX and RaB. The intensities of the lines in these series are peculiar. With the same formula the shortest possible radiation from U has been calculated and found to be of the order of magnitude of cosmic radiation.

BRYDEN[1] and White[2] have shown that the protons in the nucleus may be arranged in equivalent proton groups, which are similar to the electron configurations of the outer electrons, and that such an arrangement explains the observed nuclear moments of the first ten elements of the periodic system of elements rather well. This leads one to wonder where the electrons in the nucelus might be and what kind of equation might be obtained to express the observed γ radiations from the nucleus.

In order to study the problem the following assumptions were made. (1) Assume that the electrons in the nucleus collect to form a negatively charged center in the nucleus. (2) Assume that this center does not move. (3) Assume that the protons rotate about this center. (4) Assume that the effective charge on the center is $eZP^{\frac{1}{2}}/(t'+1)$; where P is the total number of protons present, t', with Bryden's notation, means the total quantum number of the proton which is being excited, Z takes on values of 1, 2, $\cdots$ 146 for the 1st, 2nd, $\cdots$ 146th spectrum, and e is the charge on the electron.

The fourth assumption describes the nuclear field and prescribes a screening effect of such a nature the 146 electrons may bind 238 protons in the nucleus. This assumption was stated in its present form because it leads to a formula which fits the γ-ray data to be discussed below.

By applying the simple Bohr theory to the system the total energy

$$E = -\frac{2\pi^2 e^4 Z^2 M}{h^2}\cdot\frac{P}{(t'+1)^2}\cdot\frac{1}{t^2}, \tag{1}$$

where M is the mass of proton, and t the total quantum number similar to n in the atomic case.

The frequency, expressed in cm^{-1}, radiated when a proton transition occurs thus becomes

$$\bar{\nu} = R'Z^2\frac{P}{(t'+1)^2}\left(\frac{1}{t_1^2}-\frac{1}{t_2^2}\right), \tag{2}$$

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[1] Bryden, Phy. Rev. **38,** 1989 (1931).

[2] White, Phy. Rev. **38,** 2073 (1931).

where

$$R' = \frac{2\pi^2 e^4 M}{h^3} = R\frac{M}{m}, \tag{3}$$

and R is the Rydberg constant. Thus R' is easily computed. It becomes

$$R' = 1.097 \times 10^5 \times 1836 = 2.017 \times 10^8 \text{ cm}^{-1}.$$

It also follows that the radius of an orbit is expressed by

$$A' = \frac{t^2 h^2 (t' + 1)}{4\pi^2 e^2 Z M P^{1/2}} = \frac{A}{1836} \frac{(t' + 1)}{P^{1/2}} \tag{4}$$

where $A = t^2 h^2 / 4\pi^2 e^2 Z m$, and m is the mass of the electron for the atomic case. Thus A' for gold becomes

$$A'(\text{Au}) = \frac{0.5284 \times 10^{-8}}{1836} \frac{(9 + 1)}{197^{1/2}} = 2.05 \times 10^{-12} \text{ cm}$$

where t' takes the value 9, since, according to Bryden's table, the outermost orbit in this case would be the one having 9 s^2 protons.

The only check available from experimental data on this value is one from scattering experiments. They give 3×10^{-12} cm as the distance which is the upper limit of the radius of the gold nucleus, so that the above value is a reasonable one.

A similar calculation for U gives

$$A'(\text{U}) = 2.055 \times 10^{-12} \text{ cm}.$$

A calculation of the radiated frequency (cm^{-1}) for a proton transition from the second to the first total quantum state in the U nucleus (radiation which corresponds to the $K\alpha_1\alpha_2$ x-rays of U in the atomic case) gives,

$$\begin{aligned} \bar{\nu} &= 2.017 \times 10^8 (2/2^2) 146^2 \cdot \tfrac{3}{4} \\ &= 16.14 \times 10^{11} \text{ cm}^{-1}. \end{aligned}$$

Here P is 2 since $1s^2$ protons are present before ionization and one of these $1s^2$ protons are removed from the nucleus; t' is one, since $1s^2$ protons are concerned; Z is 146.

This radiation would correspond to about 2×10^8 volts energy and would have a wave-length of 0.0618X.U. Millikan and Cameron[3] report their shortest cosmic ray to be 0.08 X.U. So it might be possible that cosmic radiation has its origin in the nucleus and is emitted by a similar process, in the nucleus, to that which produces x-ray in an atom. Moreover, it is not untenable to think that excited or ionized nuclei exist in stars. On the other hand, according to the calculations of L. H. Gray,[4] and assuming that an extrapolation of the Klein-Nishina formula is valid, observed absorption coefficients point to the existence of cosmic radiation of the order of 90 to 2,000 million volts. Such radiation could not be explained by the above formula.

[3] Millikan and Cameron, Phy. Rev. **31**, 929 (1928).

[4] L. H. Gray, Proc. Roy. Soc. **A122**, 647 (1929).

The radioactive elements fall into three groups as shown in tables on page 24 of Rutherford, Chadwick and Ellis.[5] Data from these tables have been used in substituting the proper values in formula (2) to calculate the wavelength of γ-radiation, when $t_1=1$ and $t_2=2$. The results of these calculations are given in Tables I, II and III. The observed λ's with which the calculated λ's are compared, are the longest wave-lengths for each element as given by Rutherford, Chadwick and Ellis.

TABLE I. *Comparison of observed and computed γ-ray wave-lengths for radioactive elements of group I.*

$$\tilde{\nu}=R'Z^2\frac{P}{(t'+1)^2}\left(\frac{1}{t_1^2}-\frac{1}{t_2^2}\right),\qquad t_1=1,\ t_2=2$$

Element	Type of disintegration	Total No. protons	Z	P	t'	Proton configuration	Cal. $\tilde{\nu}\times10^{8}$ cm^{-1}	Cal. λ	Obs. λ
UX_1	β	234	1	234	6	$8d^{10}9p^{6}10s^{2}6h^{14}$	7.22	138.5 X.U.	134.2X.U.
RaB	β	214	1	214	8	$7f^{14}8d^{10}9p^{2}$	4.00	250.	230.3
RaC	$\alpha\beta$	214	1	214	7*	$7f^{14}8d^{10}9p^{2}$	5.06	197.6	209.5
RaD	β	210	1	210	8	$7f^{14}8d^{8}$	3.92	255	261
Pa	—	231	1	231	6	$8d^{10}9p^{6}10s^{2}6h^{11}$	7.13	140	130.
Ra	α	226	2	225	8*	$8d^{10}9p^{6}10s^{2}6h^{5}$	16.81	59.5	65.2

TABLE II. *Comparison of observed and computed γ-ray wave-lengths for radioactive elements of group II.*

Element	Type of disintegration	Total No. protons	Z	P	t'	Proton configuration	˜ Cal $\times10^{8}$ cm^{-1}	Cal. λ	Obs. λ
AcX	α	223	2	222	10	$8d^{10}9p^{6}10s^{2}6h^{3}$	11.1	90.1X.U.	86.0X.U.
RdAc	α	227	1	227	10	$8d^{10}9p^{6}10s^{2}6h^{7}$	2.84	352.	392.
AcC	$\alpha\beta$	211	3	208	9*	$7f^{14}8d^{9}$	29.3	35.3	34.9

TABLE III. *Comparison of observed and computed γ-ray wave-lengths of radioactive elements of group III.*

Element	Type of disintegration	Total No. protons	Z	P	t'	Proton configuration	Cal. $\tilde{\nu}\times10^{8}$ cm^{-1}	Cal. λ	Obs. λ
M_sTh^2	β	228	1	228	7*	$8d^{10}9p^{6}10s^{2}6h^{8}$	5.39	186.X.U.	212.X.U.
ThD	Stable	208	1	208	9*	$6g^{18}7f^{14}8d^{6}$	3.15	317.	305.
RdTh	α	228	1	228	6	$8d^{10}9p^{6}10s^{2}6h^{8}$	7.04	142.	145.
ThB	β	212	2	211	8	$7f^{14}8d^{10}$	15.76	63.5	64.6
ThC″	β	208	1	208	9*	$6g^{18}7f^{14}8d^{6}$	3.15	317.	303.

The largest variation between observed and calculated values occurs for MsTh2 and RdAc where the difference is about 12 percent. All other cases fit reasonably well and some differences are of the order of one percent.

In column seven the proton configurations for the element in question are given. These were taken from Bryden's[1] table.

In most cases t' has the value of the total quantum number of the last sub-shell to be filled and since in general one would expect such protons to be the first to be excited, it was stated above that t' was the total quantum number of the proton being excited. In Table I, RaC and Ra are exceptions

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[5] Rutherford, Chadwick and Ellis, *Radiations from Radioactive Substances.*

to this rule and the t' values have been given an asterisk for that reason. Both of these elements emit α particles, whereas the other elements in the table emit β particles, and this may be a reason why protons of a different total quantum number, than that expected, are excited.

RaB may also seem to be an exception to the rule, but since t' is involved only in $P/(t'+1)^2$, which is a screening function, it means that the $9p^2$ protons have very little screening effect and that t' therefore has the value 8.

With this in mind Table II contains only one exception, AcC, which emits both α and β particles. Table III has three exceptions. Here t' equals the total quantum number of the last filling-shell plus one. There is no apparent reason for the discrepancy.

For the two elements AcX and RaB, it has been possible to calculate a series of wave-lengths which fit the observed wave-lengths rather well. Formula 2 has been used to make the calculations when $Z=2$, and t_1 takes on the values 3/4, 7/8, 15/16 · · · 1 successively. The calculated wave-lengths check the observed wave-lengths to less than 5 percent error.

The intensities of the γ-rays listed in Tables IV and V were obtained from β ray lines, and therefore may be in error. The limit of the AcX series is 11.10×10⁸ cm⁻¹, and is the smallest frequency of the series. The intensities, however, are degraded in the opposite direction. This presents a difficulty which can not be explained at the present time.

TABLE IV. *γ-ray spark spectrum of AcX.*

$$\tilde{\nu}=R'Z^2\frac{P}{(t'+1)^2}\left(\frac{1}{t_1^2}-\frac{1}{t_2^2}\right)\qquad \begin{array}{l} t_2=2 \\ t_1=3/4,\ 7/8,\ 15/16,\ 31/32,\ \cdots 1\end{array}$$

App. Int.	t_1	Calculated $\tilde{\nu}$	Calculated λ	Observed λ_1
130	3.4	22.6×10^8 cm^{-1}	44.2 X.U.	45.9 X.U.
25	7/8	15.63	64.0	61.7
80	15/16	13.1	76.3	78.6
180	31/32	12.0	83.0	80.6
155	1	11.10	90.1	86.0

TABLE V. *γ-ray spark spectrum of RaB.*

$Z=2$, $P=213$, $t_1=3/4$, 7/8, 15/16, 31/32 · · · 1, $t_2=2$, $t'=7$

Int.	t_1	Calculated $\tilde{\nu}$	Calculated λ	Observed λ_1
?	3/4	41.0×10^8 cm^{-1}	24.4 X.U.	26.2 X.U.
125	7/8	28.36	35.26	34.9
100	15/16	23.8	42.02	41.6
4	31/32	21.86	45.8	47.5
90	1	20.14	49.65	50.8

In Table IV the intensities decrease as the series limit is approached , except for the intensity of λ26.2 X.U. which is questionable. λ50.8 X.U. is the head of the series and may contain several lines which are not resolved. This would account for its large intensity.

Thus, although there are numerous points still to be explained, it seems more than a coincidence, that the computed and observed wave-lengths should check as well as they do. The result is that the idea of equivalent proton states in the nucleus has been supported.