The Intrinsic Parity of Elementary Particles

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The limitations to the concept of parity of quantum-mechanical states and, in particular, of intrinsic parity of elementary particles are discussed. These limitations are shown to follow from "superselection rules," i.e., from restrictions on the nature and scope of possible measurements. The existence of such superselection rules is proved for the case of spinor fields; it is also conjectured that a superselection rule operates between states of different total charge.

THE POSSIBILITY OF INDETERMINATE PARITIES

ALTHOUGH the present quantum-field-theoretic scheme to describe elementary particles is full of mathematical holes, it possesses certain features, mainly based on invariance properties, that are believed to be of far more permanent value. The importance of these features can hardly be overestimated, since they offer the most reliable guidance that we have in classifying and interpreting the rapidly growing and already very complex experimental picture.

The purpose of this paper is to point out the possible (and in certain cases necessary) existence of limitations to one of these general concepts, the concept of "intrinsic parity" of an elementary particle. Even though no radical modification of our thinking is thereby achieved, we believe that the injection of a certain amount of caution in this matter may be useful, as it may prevent one from calling "theorems" certain assumptions, or from discarding as "impossible" forms of the theory, which under a more flexible scheme are perfectly consistent. Another possible advantage of the following considerations may be to bring a certain amount of clarity in a field in which a great deal of confusion exists.[1]

The more or less standard position seems to be that every elementary particle must have a definite "intrinsic parity" factor, which can be determined unambiguously from experiment[2] (at least in principle).

In order to understand the limitations of this viewpoint, it will be useful to recall first some simple points about the formalism.

The transformation properties (in our case, the parity) of a certain kind of particles can be described in two ways; it will be useful to keep both in mind. One can state the transformation law of the quantized field. One will say, for instance, that a certain kind of spinless particles are the quanta of a "pseudoscalar" field, i.e., a field φ such that the transformation law for an inversion at the origin is

$$\varphi'(x, y, z) = -\varphi(-x, -y, -z). \tag{1}$$

Alternatively, one can state the transformation law for the state vector or Schrödinger function F, which gives the quantum-mechanical description of the state of the field,[3] i.e., one can find the unitary operator I such that

$$F' = IF \tag{2}$$

describes the state which is the mirror image of the state described by F.

The two alternative descriptions of the transformation law are, of course, very simply related, for in quantum mechanics the "observables" or operator quantities, such as the field $\varphi(x, y, z)$ above, transform according to the law

$$\varphi' = I\varphi I^{-1}, \tag{3}$$

when the state vector transforms according to (2).

Thus the unitary operator I determines completely the transformation law for the field quantities, and conversely, the transformation law for the latter is sufficient to determine the operator I. Thus, for instance, if one states that φ is a pseudoscalar, this means that

$$I\varphi(x, y, z)I^{-1} = -\varphi(-x, -y, -z), \tag{4}$$

and from this equation one may infer that I is of the form:

$$IF = \omega(-1)^{N_0+N_2+N_4+\cdots}F, \tag{5}$$

where N_l is the number of particles (of the kind described by φ) with angular momentum l while ω is an arbitrary factor of modulus unity, which remains

[1] The origin of the present article is an address which was presented by the last author at the International Conference on Nuclear Physics and the Physics of Elementary Particles in September 1951 in Chicago and which was based on a review article which the last two authors are preparing together with V. Bargmann. In view of several inquiries concerning the above-mentioned address, the authors feel that a preliminary publication of some of the main points in the present paper is justified, even though they must refer the reader to the review article to appear later for a more exhaustive and consistent presentation of the whole subject.

[2] This is no doubt an oversimplified version even of "current" belief, especially in the case of spin $\frac{1}{2}$ particles. This case, however, will be discussed later in greater detail.

[3] In this particular kind of discussion it seems to be a good idea to avoid the chameleon-like term wave function. We shall adhere strictly to state-vector in one case and field function in the other.

indeterminate in any quantum mechanical transformation. If one arbitrarily sets $\omega=1$, the vacuum vector F_0 will satisfy

$$IF_0=F_0, \tag{6}$$

i.e., the vacuum will be an "even" state. A one-particle S-state will be "odd," etc. It is, of course, the presence of even angular momenta in the exponent of (5) which characterizes "pseudoscalar" particles, rather than the arbitrary choice of even and odd states which is conventionally determined by (6). This is, however, a convenient choice which is usually made.

Similarly, for the particles of a scalar field one finds

$$IF=(-1)^{N_1'+N_3'+N_5'+\cdots}F, \tag{5a}$$

where the N' have the same significance for the scalar particles as the N had for the pseudoscalar ones. If F describes an aggregate of pseudoscalar and scalar particles, one has, naturally

$$IF=(-1)^{N_0+N_2+\cdots+N_1'+N_3'+\cdots}F. \tag{5b}$$

We can now resume the discussion of the main point. The customary assumption is that the field quantities must obey an unambiguous transformation law, or alternatively, in the quantum mechanical case, that the unitary operator I is well defined, apart from the trivial factor ω. Once that is granted, it follows for instance that a scalar field must be either a "true" scalar or a "pseudo" scalar. Such assumptions are even believed to be a logical necessity, if symmetry operations, such as the inversion, are to have a well-defined meaning at all.

Nevertheless, everybody knows that the transformation law of spinor quantities, such as the Dirac field ψ is not unambiguous. Yang and Tiomno, in their interesting paper[4] on the inversion law for spin 1/2 fields, which has been the origin of much thinking on this matter, do in fact assume an essential ambiguity in the sign of $I\psi I^{-1}$. While the case of spin 1/2 particles offers perhaps the best substance to our doubts, it is preferable to state first our position in a quite general manner.

In our opinion, the whole question hinges on what one can say about the measurability of the field operators. In fact, if a field is measurable, then it must have a well-defined expectation value in any given state. The situation is then equivalent to the classical one, where one can argue that there must be a definite transformation law, just because the fields are regarded as well-defined physical quantities. If, however, a field quantity is not measurable—and, as we shall see below on the example of the Dirac field, there are such fields—there is no logical need for an unambiguous transformation law.

It is quite true, of course, that the assumption that "all Hermitean operators represent measurable quantities" is often presented as an integral part of the general scheme of quantum mechanics. It is also true that in the case of the ordinary nonrelativistic quantum mechanics of particles this assumption, implausible as it sounds for all but the very simplest operators, is not subject to any very serious objection. It should be hardly necessary to point out, however, that a wholesale extension of the measurability postulate to the physical abstractions with which the present field theory of "elementary" particles operates is an unwarranted and enormous extrapolation, especially in view of our scarce knowledge of the actual interaction laws.

That no intrinsic difficulties are inherent in such a position, will be apparent if one considers that it is perfectly possible to construct logical schemes in which the unrestricted measurability postulate is abandoned. It is indeed surprising, in view of the known nonmeasurability of the Dirac field, that the nature of such schemes has not been more widely discussed.

What we have in mind is the following. The usual assumption in quantum mechanics is that it is possible to carry out a "complete" set of measurements, the result of which determines the state vector F completely, except for the usual phase factor. Suppose now instead that the Hilbert space can be decomposed into certain orthogonal subspaces $A, B, C, \cdots$ such that the relative phase of the components of F along $A, B, C, \cdots$ is intrinsically irrelevant. In other words, calling these components (which are themselves vectors) F_a, F_b, F_c, $\cdots$, assume that no physical measurement can distinguish between the state-vectors

$$F_a+F_b+F_c+\cdots \tag{7}$$

and

$$e^{i\alpha}F_a+e^{i\beta}F_b+e^{i\gamma}F_c+\cdots, \tag{7'}$$

where $\alpha, \beta, \gamma, \cdots$ are arbitrary phases. It is clear then, that the expectation value of any operator that has matrix elements connecting the subspaces A and B, or A and C, etc., will be, in general, completely undefined. Hence such an operator will not correspond to a measurable quantity. Such an assumption is not incompatible with the other rules of quantum mechanics, and in particular with the superposition principle; a linear relationship between vectors will remain unaltered if all vectors simultaneously are subjected to the transformation (7′). One may indeed regard this transformation as a generalization of the ordinary multiplication of the whole vector by one phase factor.

Another, and more familiar, way of describing this situation is to say that the state described by (7′) is not a pure state, but a statistical mixture, which could be best described by a density matrix.[5] The assumption presented above is that such a density matrix represents the maximum possible amount of knowledge. The system can, of course, be in a pure state in the ordinary sense, but only if only one of the components, say F_a,

[4] C. N. Yang and J. Tiomno, Phys. Rev. **79**, 495 (1950); see also E. R. Caianiello, Nuovo cimento **7**, 534 (1951); **8**, 749 (1951); **9**, 336 (1952).

[5] For this concept see J. v. Neumann, *Mathematische Grundlagen der Quantenmechanik* (Verlag. Julius Springer, Berlin, 1932), Chap. IV.

is finite. If the above is admitted, the unitary operator I representing a symmetry operation will be affected by a corresponding lack of definition. Ordinarily I will have no matrix elements connecting the subspaces A, B, C, $\cdots$; it will leave each invariant. Hence the submatrices of I in any of the subspaces A, B, $\cdots$ will contain an arbitrary factor ω_a, ω_b, $\cdots$ just as the whole I contained such an arbitrary factor. Because of the equivalence of the wave functions (7) and (7′) there will be no way to determine the ratios of ω_a, ω_b, $\cdots$; instead of the single indeterminate phase factor there will be as many as there are subspaces A, B, $\cdots$. This means that it will not be possible to make any statement as to the relative parity of states belonging to different subspaces.

It is customary to say that a selection rule operates between subspaces of the total Hilbert space if the state vectors of each subspace remain orthogonal to all state vectors of the other subspaces as long as the system is isolated. There is, for instance, a selection rule which prevents any state of an isolated system from changing its total linear momentum. Similarly, the state vectors of the subspace containing all states with total angular momentum J will remain, in a closed system, orthogonal to all states with any other total angular momentum. We shall say that a superselection rule operates between subspaces if there are neither spontaneous transitions between their state vectors (i.e., if a selection rule operates between them) and if, in addition to this, there are no measurable quantities with finite matrix elements between their state vectors. This is the situation described above; it entails that the phase factors ω_a, ω_b, $\cdots$, given above, are all unobservable. The new point which we wish to bring out is that there is definite evidence that such superselection rules exist in the present formalism of relativistic field theories. We shall outline a proof for this in one case and will suggest another case in which it is also very likely to operate.

The existence of superselection rules allows one more freedom than one would perhaps like to have. We are not especially concerned here, however, with the possibility of exploiting this freedom to the utmost limit,[6] in order to produce "monsters" with unexpected properties. Rather, we are interested in the most simple instances in which the above described situation appears to prevail. It would be quite wrong to assume that a superselection rule operates, for instance, between subspaces with different total linear momenta. The phase between such states is measurable, and every position measurement, in fact, involves the measurement of phases between states of different linear momenta.

[6] Strictly speaking, even if one assumes a definite transformation law, one cannot exclude on general grounds a more complicated transformation, such as $\varphi'(x, y, z) = \omega(N)\varphi(-x, -y, -z) \times \omega(N)^{-1}$ where the value $\omega(N) = \pm 1$ can be chosen at will for every N irrespective of the product rule. This possibility can be easily excluded only if φ is a locally measurable quantity.

SPINORS

One must introduce a superselection rule between at least two subspaces of the whole Hilbert space if one wishes to preserve the relativistic invariance of this space. The first of these subspaces, A, contains the states in which the total angular momentum of the system is an integer multiple of $\hbar$, the second subspace B contains the states with half-integer angular momenta. Let us denote the state vectors of the first subspace by f_A, g_A, $\cdots$, those of the second by f_B, g_B, $\cdots$. We shall consider states $2^{-\frac{1}{2}}(f_A+f_B)$ for which the measurement of the angular momentum gives with a probability 1/2 an integer angular momentum and with the same probability a half-integer angular momentum. Let us imagine, furthermore, that the two states $2^{-\frac{1}{2}}(f_A+f_B)$ and $2^{-\frac{1}{2}}(f_A-f_B)$ can be distinguished by some measurement. This is what we mean by the statement that the phase between the subspaces A and B can be measured. We shall see that this assumption cannot be reconciled with the requirement of relativistic invariance.

Our proof for this will be based on the transformation of time inversion. This transforms f_A into U_AKf_A and f_B into U_BKf_B in which the U are unitary operators and the K indicates that one has to take the conjugate complex of the ensuing expression. The crucial point of our proof is based on the equations[7]

$$U_AKU_AK=1; \quad U_BKU_BK=-1. \tag{8}$$

Naturally, U_AK and U_BK can be replaced by ωU_AK and $\omega' U_BK$ without changing the content of the theory as long as $|\omega| = |\omega'| = 1$. Such a substitution, however, will leave (8) unaffected. It is this circumstance which renders a proof based on the transformation of time inversion particularly simple.

Applying the operation of time inversion to a state f_A+f_B will give $\omega U_AKf_A+\omega' U_BKf_B$ in which ω is, of course, indeterminate but ω'/ω, though unknown, will be independent[8] of the state vectors f_A and f_B. Applying now the operation of time inversion again, we must obtain a state which is indistinguishable from the original f_A+f_B. The result is

$$\omega'' U_AK(\omega U_AKf_A) + \omega''' U_BK(\omega' U_BKf_B), \tag{9}$$

with $\omega'''/\omega'' = \omega'/\omega$. Because of this and (8), (9) becomes

$$\omega''\bar{\omega}U_AKU_AKf_A+\omega'''\bar{\omega}'U_BKU_BKf_B = (\omega''/\omega)f_A-(\omega'''/\omega')f_B = \text{const}(f_A-f_B). \tag{9a}$$

In view of the different signs in (8), this result was to be expected; it shows that f_A+f_B and f_A-f_B must remain indistinguishable as long as a time inversion operator satisfying (8) exists. The same result could have been obtained by considering rotations instead

[7] See E. Wigner, Nachr. Ges. Wiss. Göttingen, p. 546 (1932).

[8] This is a crucial point which, however, has been discussed repeatedly. See E. Wigner, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren* (Friedr. Vieweg. Braunschweig, 1931), Appendix to Chapter XX. A simplified proof will be given in the article mentioned in reference 1.

of the time inversion. However, the consideration becomes somewhat more involved because there is no equation for rotational transformations, similar to (8), which would remain unchanged if one replaces the transformation by a multiple thereof.

It follows from the above that the measurability of any Hermitean operator ξ which has finite matrix elements $(f_A, \xi f_B)$ between the subspaces A and B (i.e., between states with integer and half-integer angular momenta) would lead to a contradiction. In fact, unless $(f_A, \xi f_B)$ is purely imaginary, the expectation values of ξ for the states f_A+f_B and f_A-f_B would be different. However, we have seen that these states are undistinguishable. If $(f_A, \xi f_B)$ is purely imaginary, the above statement applies to the pair of states f_A+if_B, f_A-if_B which can be shown to be similarly undistinguishable.

Since every spinor field ψ has the property that both $\psi+\psi^*$ and $i(\psi-\psi^*)$ connect the subspaces A and B, it follows that neither of these two quantities can be measurable (ψ itself is not Hermitean and, for this reason, its measurement need not be considered).

CHARGED FIELDS

In the present form of field theory, charged particles are represented by complex fields. If only one such field is considered, say $\varphi(x, y, z, t)$, the Lagrangean and Hamiltonian functions, including if necessary the interaction with external fields, will contain φ only in the bilinear combination $\varphi^*\varphi$, i.e., they will be invariant against multiplication of φ by a phase factor $e^{i\alpha}$. We are thus led to believe that such a factor is an intrinsically unobservable modification of the field. If several charged fields $\varphi_1, \varphi_2, \cdots$ be present, the Hamiltonian may contain terms such as $\varphi_1{}^*\varphi_2$, $\varphi_1{}^*\varphi_2{}^*\varphi_3{}^2, \cdots$ etc., but in all cases it is invariant against a simultaneous multiplication of all fields by the same $e^{i\alpha}$.

This property is known to be connected with the principle of conservation of the total charge and represents a very restricted type of gauge invariance. If Q is the total charge, in terms of e as a unit, multiplication of $\varphi_1, \varphi_2, \cdots$ by $e^{i\alpha}$ can be achieved by the unitary transformation

$$\varphi_s \rightarrow e^{-i\alpha Q}\varphi_s e^{i\alpha Q}. \tag{10}$$

We are thus led to postulate that: multiplication of the state vector F by the operator $e^{i\alpha Q}$ produces no physically observable modification of the state of a system of (mutually interacting) charged fields.

We can give no conclusive evidence for this assertion, and such evidence may in fact depend on a deeper understanding of the meaning of electric charges which we still lack. Assuming that the assertion is correct, it follows that the parities of states with different charges cannot be compared.

As a result, it is clear, for instance, that if certain experimental data can be interpreted on the assumption that the charged π-meson field is pseudoscalar and the other (proton, μ-meson, $\cdots$ etc.) charged fields with which it interacts have certain specified inversion properties, it must be equally possible to interpret the data on the assumption that the charged π-meson field is scalar, provided corresponding modifications in the properties of the other fields are made.[4]

APPLICATIONS

Having stressed purely negative aspects so far, let us see what one can say in an affirmative sense.

In the first place, the electromagnetic field is no doubt the one about which we know most. Once it is stated that the electric field is a polar vector,[9] one knows the properties of any state containing only photons.

The parity of a particle, like the neutral π^0, which can decay into a pure photon state, is then in principle determinable. Another way to do this is to ascertain which selection rules obtain experimentally in a reaction such as $p+p\rightarrow\pi^0+p+p$, where no other particle is created or destroyed.

If we turn now to charged particles, our considerations show that parities are to a certain extent arbitrary. This means, as it often happens, that we need a frame of reference, which is based on conventions, but is no less useful because of that. We could, for instance, agree that the $\pi^\pm$ mesons are to be regarded as odd. This would then reduce the arbitrariness in the inversion law for other particles. For instance, the well-known capture experiments in deuterium give indications as to parity, that can be formulated in the above frame of reference, by saying that the proton and neutron fields ψ_P and ψ_N transform in such a way that $\psi_P{}^*\beta\psi_N$ is a scalar.

Leaving these special examples, it is perhaps desirable to state in general what possibilities exist; this will also clarify the difference as well as the area of agreement between our standpoint and that adopted in the paper of Yang and Tiomno.[4]

The preceding remarks should demonstrate that the possibility of determining or comparing intrinsic parities is intimately connected with the possibility of performing quantum mechanical experiments which can serve to determine phase differences between different parts of a state function. If there were no superselection rules, i.e., if all phase differences could be measured in principle, the relative parities of all particles could be determined. This could be done in prin-

[9] It is, of course, true that if, in addition to the inversion I as it is usually understood, (i.e., such that the sign of the charges is preserved, and E is polar) one believes in the operation of charge conjugation C as an exact symmetry property of nature it becomes arbitrary whether one regards I or CI as the inversion law. Adopting CI, however, none of the states of atoms or nuclei, that one considers normally, would be a state of definite parity (states of definite parity would involve superpositions of states containing protons and anti-protons, etc.). The definition thus is not a convenient one to adopt. That C is an exact symmetry property is moreover still far from proved. The disturbing possibility remains that C and I are both only approximate and CI is the only exact symmetry law. This would force us to regard the electric field as an axial vector. This possibility, however, seems rather remote at the moment.

ciple, for instance, by constructing a state which is, with probability 1/2, a particle A with angular momenta J and J_z about a point and a line, with probability 1/2, another particle B in a similar state. If this state looks the same in a mirror which is parallel to the line, and if rotated by π about an axis perpendicular to the line, the parities of A and B are the same; they are opposite otherwise.[10] Less abstractly, one could try to transform A into B and keep track of the parities of the particles which were absorbed and emitted during the transformation.

The superselection rule which prevents the comparison of phases between states with half-integral and integral angular momenta makes it impossible to compare directly the parities of spinor particles with those of integral spin particles. However, if this is the only superselection rule, it remains possible to compare the joint parities of two identical particles with that of an integral spin particle. If this parity be even, one would be tempted to attribute a real parity to the individual spinor particle. If the parity is odd, one would be tempted to attribute an imaginary parity thereto. However, still proceeding on the assumption that the spinor superselection rule is the only one in existence, and that all phases can be measured, the measurement of which is not forbidden by this superselection rule, the parities of any two spinor particles can be compared. This comparison, as any comparison of parities, can yield only the results "equal" or "opposite." Hence, if one spinor particle had a real parity in the above sense, this will be true for all other spinors. Similarly, if one spinor had an imaginary parity, this will be true of all others. Less abstractly, some pairs of spinors may disintegrate in such a way that one will say that the product of their parities is even, some pairs in such a way as to make the product of their parities appear odd. Under the assumptions of this paragraph the product of their parities clearly cannot be imaginary because every pair will be able to transform into integral spin particles, possibly after absorbing some such particles. In particular, under the assumption of this paragraph, it would not be possible for one pair of identical spinors s_1 to show even, for another pair s_2, to show odd parity. In this case a pair of s_1 plus a pair of s_2 would show odd parity. This would mean, however, that the parity of a pair consisting of one s_1 and one s_2 particle would have to be indeterminate. This is contrary to our assumption.

If we assume that the phases of states with different charges cannot be compared either, i.e., assume a superselection rule for charge, there will be no direct way to compare parities of particles with different charges. However, it will be again possible (unless prevented by the spinor superselection rule) to compare the product of the parities of two particles with opposite charges, with the parity of an uncharged particle. This parity can again be the same or opposite. Since an identical pair of charged particles cannot be uncharged, the square of the parity of a charged particle cannot be determined under the present assumption. It will be possible to say that the parity of a particle of unit charge is ω, where ω may be any number of modulus 1. Still under the assumption of this paragraph, every other particle with unit charge (and the same type of spin) will then have parity ω or $-\omega$, every particle with opposite charge the parity ω^{-1} or $-\omega^{-1}$. Clearly, it will not be possible to attribute a direct physical significance to the quantity ω and one may just as well call it 1. However, there may be some advantage in keeping ω because it will remind one of the conservation law for electric charges. An interaction operator which violates this law will also appear to violate the principle of inversion symmetry. If one assumes the conservation law for heavy particles to hold and adopts a corresponding superselection rule, forbidding the observation of phase differences between states with different numbers of heavy particles, the parity of a heavy particle will have a new indeterminate factor ω' in it which again will have no immediate physical significance. Similar remarks apply to ω' as were made above for ω. It may be again desirable to keep this indeterminate phase as a useful formal device to remind one of this conservation law.

A slightly less general device is that employed by Yang and Tiomno, who by a suitable choice of factors, $\omega' = \pm 1$, $\pm i$ for the various particles, succeed in excluding many of the interactions that one would otherwise be tempted to assume and which are in contradiction with the conservation law for heavy particles.

[10] It is hardly necessary to point out that within a subspace A, or B, . . . the operator I^2 must be a multiple of unity, for the customary reasons; see, for example, reference 7. Hence the parities of two states, whenever they can be compared, can only be equal or opposite.